Computer Networking and IT Security (INHN0012)

Tutorial 3

Problem 1 Probability of frame errors

The lecture introduced bit error probabilities for radio links as about $p_{e,1} = 10^{-4}$ and for Ethernet over copper cable as about $p_{e,2} = 10^{-8}$. We assume that bit errors occur independently and uniformly distributed through noise with constant power over time. Thus, the channel characteristics do not change over time. For simplicity, we omit other interfering factors. Let the frame length be equal 1500 B.

a)* For both types of transmission, determine the probability that a frame will be transmitted without errors.

$$\Pr[\text{„no bit-error in frame"}] = (1 - p_e)^{\ell \cdot 8}$$

For wired connection this results in a probability of success of 99.99 %, for wireless transmission only 30.12 %.

From here on, we will only consider the wireless connection. Since the frame error probability is relatively high, a protocol on the link layer provides acknowledgements for correctly transmitted frames. If no acknowledgement is received for a previously sent message, the sender assumes that the transmission was not successful. For the sake of simplicity, we assume that confirmations are not lost.

b)* Is there a maximum number of repetitions until a certain frame is guaranteed to have been transmitted correctly?

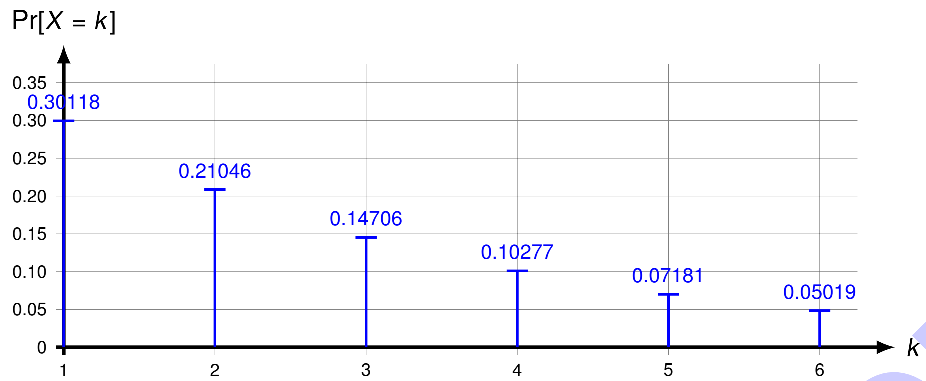
No. The individual transmissions are independent of each other, i.e. a frame error with the probabilities calculated in a) also occurs with each repetition. So there is always a residual risk.

c)* Determine the probability that the frame must be transferred exactly k times.

The random variable X indicates the number of necessary transmissions.
With $p_R = \Pr[\text{„no bit-error in frame"}]$ results:

$$\begin{aligned}\Pr[X = k] &= \Pr[\text{„}(k-1)\text{ unsuccessful transmissions "}] \cdot \Pr[\text{„no bit-error in frame"}] \\ &= (1 - p_R)^{k-1} \cdot p_R\end{aligned}$$

d)* Sketch the probability from subtask c) for $k \in \{1, ..., 6\}$.



e) Assume that the responsible protocol on the link layer aborts the retransmission if the third transmission attempt was unsuccessful. What is the probability that a frame cannot be transmitted?

The probability corresponds to the transmission failing three times in a row without regard to whether it works the 4th time or not. This results to

$$\Pr[X > 3] = 1 - \Pr[X \leq 3] = 1 - \sum_{k=1}^{3} \Pr[X = k] \approx 34\,\%$$

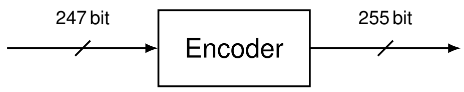
Alternative solution:

$$\Pr[X > 3] = (1 - p_R)^3 \approx 34\,\%$$

Attention: The alternative solution is only correct because X is geometrically distributed and the geometric distribution is memoryless, i.e. the failure of the k-th transmission does not influence the $(k+1)$-th transmission. If this independence were not fulfilled, the alternative solution would give a wrong result!

Problem 2 Channel Coding

In the previous task we saw that the frame error probability can become a problem with poor channel quality. For the radio channel with a bit error probability $p_e = 10^{-4}$, the success probability for a frame of length 1500 B was only about 30 %. To counter the high bit error rate, a block code is now used on Layer 1:



This allows the decoder on the receiver side to correct *any* bit error in a channel word of length $n = 255$ bit. If two or more bit errors occur, the decoder's decision is wrong and all the information of the channel word is lost.

a)* Determine the code rate.

$$R = \frac{k}{n} = \frac{247}{255} \approx 0.97$$

b)* What does the code rate express?

The code rate expresses the ratio between the size of a user data block and the size of a user data block (channel word) secured by redundancy. The smaller R, the more redundancy was added. Thus, for $R = 247/255$, each channel word of length 255 bit carries a total of 8 bit of redundancy as well as 247 bit of information.

c)* Since the frame is larger than a block of 247 bit, it must be divided into several blocks. Determine the number N of channel words that must be transmitted.

Each channel word of length 255 bit carries 247 bit user data. The result is therefore:

$$N = \left\lceil \frac{1500 \cdot 8}{247} \right\rceil = 49.$$

d) Padding is included in the last channel word. Determine the percentage overhead of the padding in relation to the possible user data in the channel words.

maximum possible payload = $N \cdot 247\,\text{bit} = 12\,103\,\text{bit}$.
padding = possible payload - actual payload = $12\,103\,\text{bit} - 1500\,\text{B} \cdot 8 = 103\,\text{bit}$

$$\gamma = \frac{\text{padding}}{\text{maximum possible payload}} = \frac{103\,\text{bit}}{12\,103\,\text{bit}} \approx 0.85\,\%.$$

Alternatively:

$$\gamma = \frac{\text{Padding}}{\text{actual Payload} + \text{Padding}} = \frac{103\,\text{bit}}{1500\,\text{B} \cdot 8 + 103\,\text{bit}} \approx 0.85\,\%.$$

e)* Determine the probability that a single channel word is decoded incorrectly.

The probability that a single channel word is decoded with errors corresponds to the probability that two or more errors occur within the channel word. Let X be the random variable indicating the number of bit errors in a channel word of length n.

$$\begin{aligned} p_{e,codeword} &= \Pr[X \geq 2] = 1 - \Pr[X \leq 1] = 1 - \sum_{i=0}^{1} \binom{n}{i} \cdot p_e^i \cdot (1 - p_e)^{n-i} \\ &= 1 - \left(1 \cdot p_e^0 \cdot (1 - p_e)^{255} + 255 \cdot p_e^1 \cdot (1 - p_e)^{254}\right) \\ &\approx 1 - \left(0,9748 + 255 \cdot 10^{-4} \cdot 0,9748\right) \\ &\approx 3.18 \cdot 10^{-4} \end{aligned}$$

f) Now determine the probability that a frame will be transmitted correctly — that is, none of the channel words that make up the frame will be transmitted incorrectly.

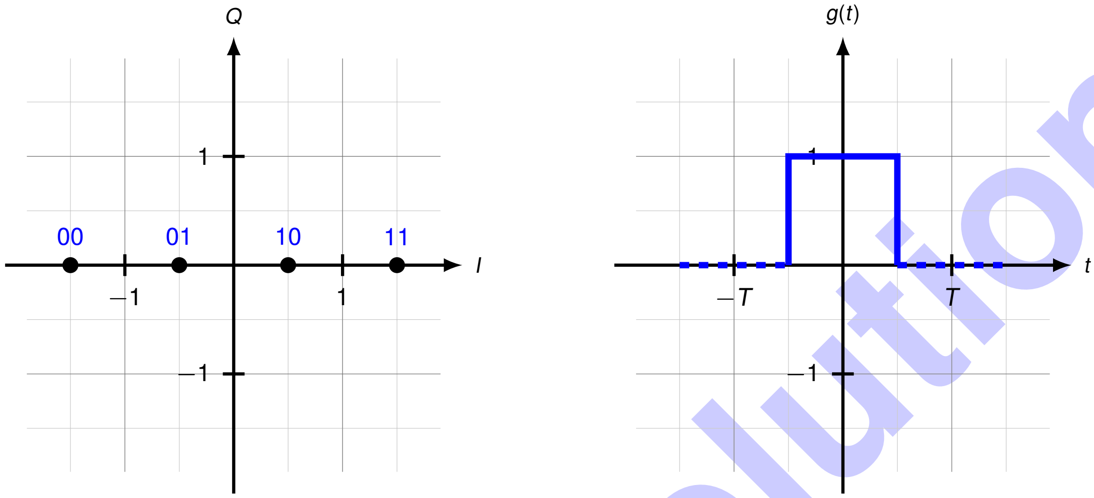
For the frame to be transmitted correctly, all channel words must be correctly be transmitted correctly. With the results of the previous subtasks therefore:

$$\Pr[\text{„no error in frame“}] = (1 - p_{e,codeword})^N \approx 98.50\,\%$$

Problem 3 Digital Modulation Schemes

In this exercise, we analyze the processes of pulse shaping in the baseband and the subsequent modulation. Figure 3.1 shows the signal space of a digital modulation scheme. The bit sequence to transmit is `01111001`. As the basic pulse for the baseband signal, the rectangular pulse rect(t) is used.

$$\mathrm{rect}(t) = \begin{cases} 1 & -T/2 \leq t < T/2 \\ 0 & \text{otherwise} \end{cases}$$



(a) Signal space

(b) Transmit pulse

Figure 3.1: Signal space and transmit pulse

a)* Which modulation scheme is shown?

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b)* Assign valid codewords to the signal points in Figure 3.1a.

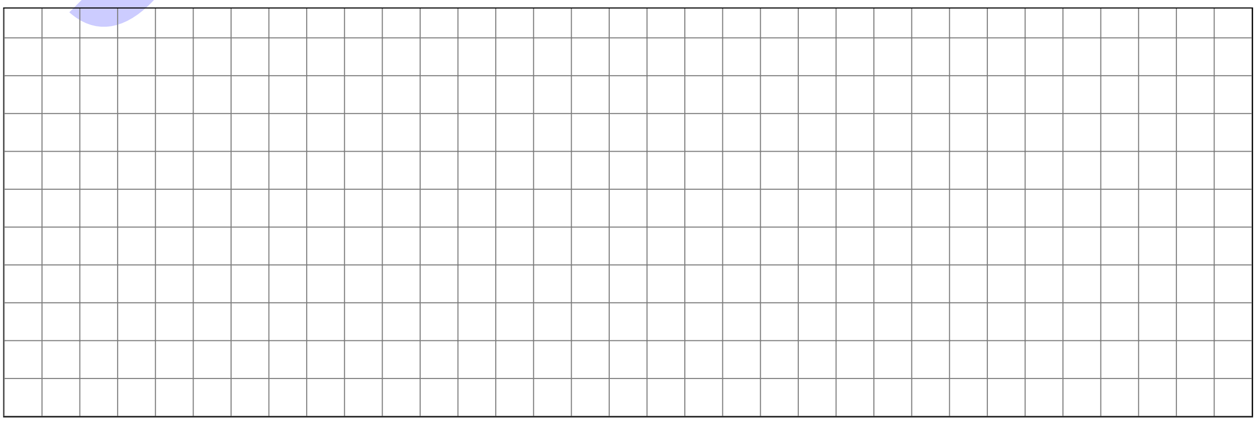
c)* Sketch the transmit pulse $g(t)$ in Figure 3.1b.

d)* Now sketch the corresponding baseband signal for the given bit sequence in Figure 3.2.

The baseband signal from the previous subproblem is now used to modulate the cosine carrier

$$s(t) = \cos(2\pi t/T).$$

e) Now sketch the modulated signal in Figure 3.2.



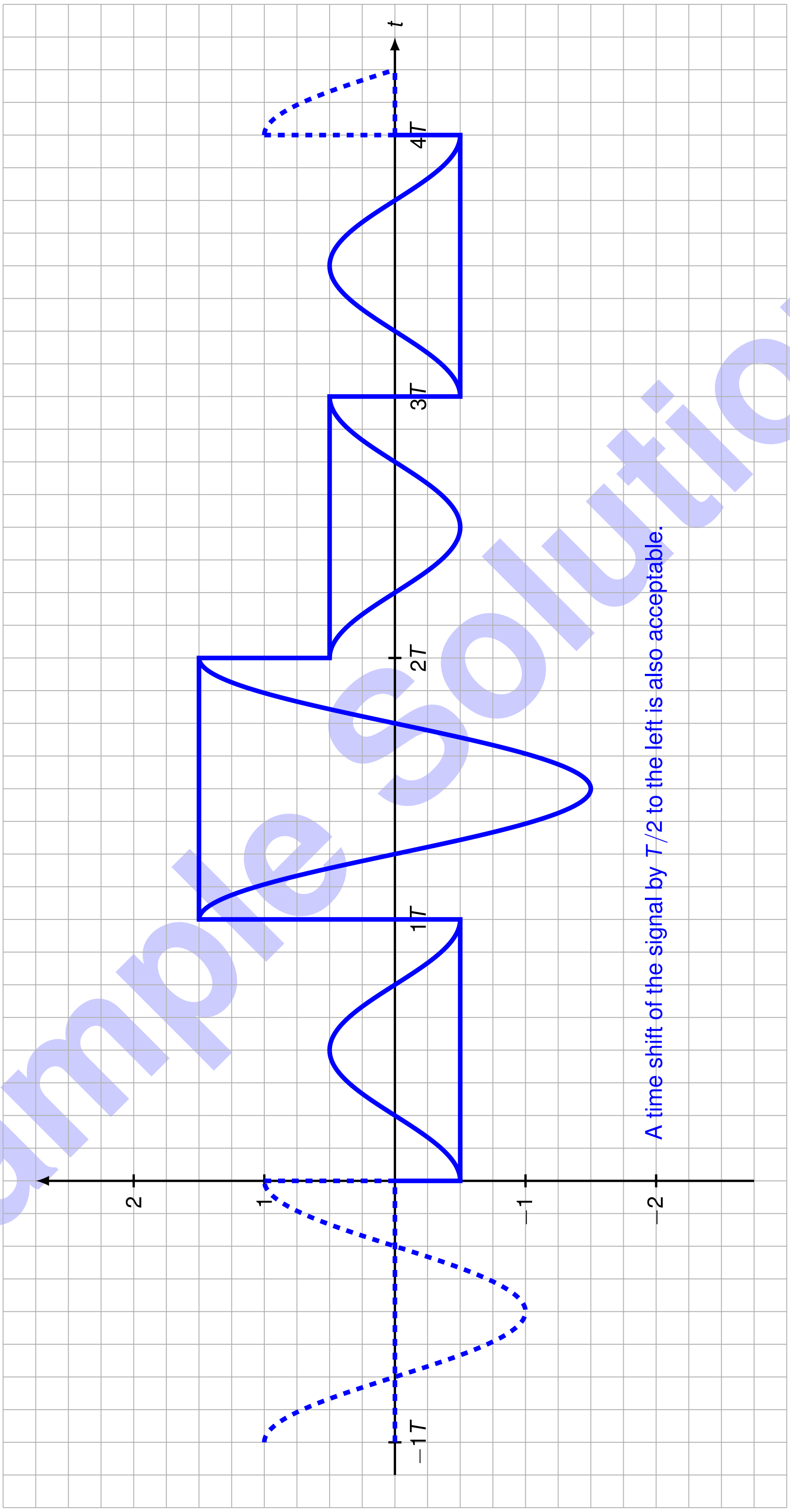


Figure 3.2: Solution sheet for subproblems d) and e)